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Singularities of dynamic stress in angular points of elastic prism of compound rectangular cross-section

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A knowledge of the nature of the behavior of the components of the stress-strain state close to singular points and lines of the surface of the body being considered enables one to approximate the solution of problems in the theory of elasticity in an optimal manner and to construct an efficient numerical algorithm in order to find it. This undertaking is even more urgent in problems of the vibration loading of structural components when the stressed state can undergo qualitative changes depending on the frequency of the external load. The discontinuities in the distribution of the static stresses in the neighborhood of the corner point of the line of separation of the domains of the cross-section of a body, composed of two different prismatic bodies which have been joined along a lateral surface, have been considered earlier in [1, 2, 3]. For example, the plane, electrostatic problem of two unlike wedges with arbitrary aperture angles was considered in [1]; the solution was constructed in terms of Mellin transforms which, after satisfying the matching conditions, enables one to investigate the dependence of the order of the singularity of the stress field at the vertex of the wedges on the aperture angles and combinations of the constants of elasticity. A method was described in [4] which enables one to establish the nature of the above-mentioned discontinuities without solving the boundary-value problem directly. Dynamic aspects of the problem have been considered in [5, 6, 7] and, in particular, the concept of a "boundary" resonance, which is a generalization of the thoroughly investigated edge resonance [8], was introduced.

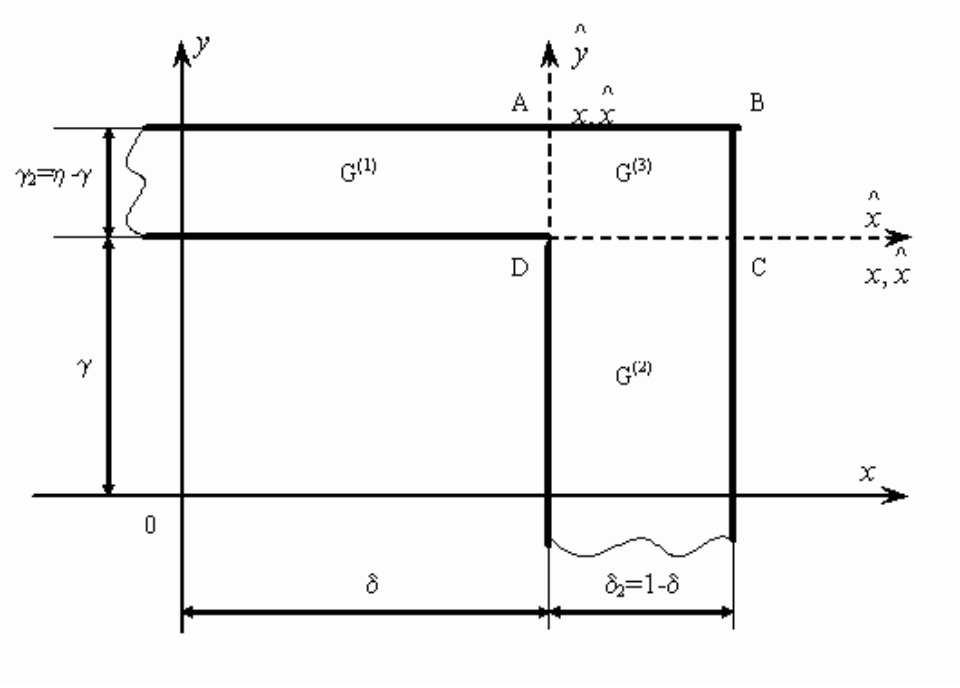

Figure 1: The region under considering in dimentionless coordinates

The problem of determining the qualitative and quantitative nature of the discontinuity in the wave

field, which arises in the neighborhood of the corner point of the joint of three unlike regions of rectangular form (see Figure 1), is presented below. Such problems arise when calculating the strength parameters of welded or soldered butt joints with angular joints [6].

In accordance with the algorithm for the modified superposition method, which was proposed firstly for the case of homogeneous, finite regions [9] and a domain extended into inhomogeneous domains [6], we replace the initial boundary conditions. This enables us to obtain an analytical solution of the subsidiary problems. The solution of the initial boundary-value problem will be expressed in terms of additional functions, which specify the boundary conditions which have been introduced. After replacing the initial boundary-value problem by the subsidiary problem, defined by boundary conditions. They can be considered as a system of integral equations in the unknown subsidiary functions. These functions can have singularities at the edge points of their domains of definition. By taking account of these singularities we can separate out and sum slowly converging parts in the series for the wave characteristics and to successfully select the coordinate functions in asymptotic methods for solving a system of integral equations. In the numerical analysis of problems of the type being considered, the main attention is given to investigation the spectrum of resonance frequencies and the maximum dynamic stresses. However, a numerical investigation of the of the problem with the aim of determining the parameter for the local singularity at the internal corner point of the section is also of interest.

CONCLUSIONS

The determination of the real roots of characteristic equation enables one to predict the nature of the dynamic concentration of stresses in the unsafe zones of a section of prismatic composite solids. By selecting the elasticity characteristics of the joined regions, which correspond to the maximum values of the local singularity parameter, it is possible to minimize the dynamic stresses at the singular points of the boundary of a section, which correspond to the internal corner points of the joint of unlike materials. The result obtained can be used when designed welded, soldered and glued corner joints operating in a vibration field.

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